Observation of generalized synchronization of chaos in a driven chaotic system

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We report on the experimental observation of the generalized synchronization of chaos in a real physical system. We show that under a nonlinear resonant interaction, the chaotic dynamics of a single mode laser can become functionally related to that of a chaotic driving signal and furthermore as the coupling strength is further increased, the chaotic dynamics of the laser approaches that of the driving signal. [S1063-651X(98)12004-4]

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I. INTRODUCTION

Due to the theoretical significance and potential practical applications, synchronization of chaos between coupled chaotic systems has been an active topic of research recently [1–8]. Synchronization of chaos was initially understood to be the perfect coincidence of the chaotic dynamics of two coupled chaotic systems. This kind of synchronized chaos has actually been observed experimentally between coupled identical chaotic systems [4–6]; however, it is now recognized that this form of synchronization is only a particular case and, in fact, other, more complicated forms of synchronization exist. Based on a drive-response system, Rulkov *et al.* [9] have proposed a generalized concept of synchronized chaos. According to them, the dynamics of two coupled chaotic systems are synchronized if a functional relationship between their dynamical variables exists. This synchronization of chaos is called the ''generalized synchronization'' of chaos, and the former one ''complete synchronization'' of chaos. Kocarev and Parlitz [10] have studied the conditions for the occurrence of generalized synchronization. They have shown that in the generalized synchronization the response is predictable, and the chaotic dynamics of the response system is conditionally equivalent to that of the driving system.

Although generalized synchronization of chaos has been theoretically investigated, and it can occur between coupled nonidentical chaotic systems, so far it has only been experimentally observed in a system of coupled nonlinear electronic circuits [9]. In this paper we report on another experimental observation of the generalized synchronization of chaos. We show that the chaotic intensity dynamics of a single mode laser can be synchronized in the sense of generalized synchronization to that of a chaotic driving signal. On the basis of our experimental results, we also explain the relationship between the generalized synchronization of chaos and the complete synchronization of chaos.

II. EXPERIMENTAL SETUP

Our experiment was carried out on a chaotically driven chaotic system. A schematic block diagram of our experimental system is shown in Fig. 1. The chaotic system used in our experiment is the optically pumped $^{15}NH_3$ single mode ring laser operating chaotically. Details about the laser's configuration can be found in Ref. [11]. The chaotic dynamics of the laser without chaotic driving have been intensively investigated before, and deterministic chaos such as period-doubling chaos, Lorenz spiral chaos, and intermittent chaos are typical chaotic dynamics of the laser [12–14]. Under certain conditions the chaotic dynamics of the laser are well described with the single mode laser Lorenz-Haken equations [15,16]. In the present experiment the laser is operated in the parameter regime where the laser behaves closely to the dynamics of the laser Lorenz-Haken equations. Our experiments correspond to driving the Lorenz control parameter r with a chaotic signal. An acousto-optic modulator (AOM) is used to modulate the pump intensity of the laser. Experimentally, in order to obtain a controllable chaotic driving signal, the following procedure was used to create the chaotic pump modulation. First, we either recorded a chaotic intensity wave form of the laser without chaotic driving or we calculated a chaotic wave form from the Lorenz equations. This chaotic wave form was then stored in the memory of an arbitrary function generator (AFG). Based on the stored chaotic wave form, the AFG produces an analog signal that has exactly the same wave form as that of the stored one, but its amplitude and average chaotic pulsation frequency are continuously tunable. This analog signal is then used to amplitude modulate the rf driving signal of the
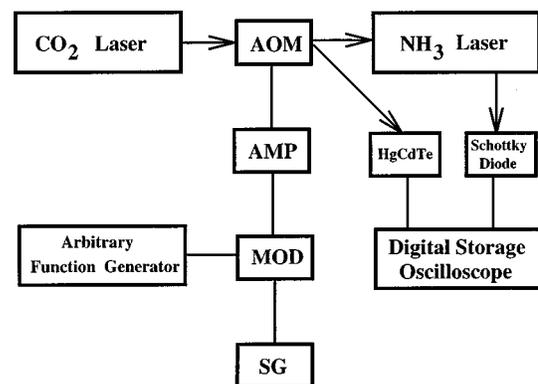

FIG. 1. Schematic block diagram of the experiment setup. SG, signal generator; MOD, modulator; AMP, amplifier; AOM, acousto-optic modulator.

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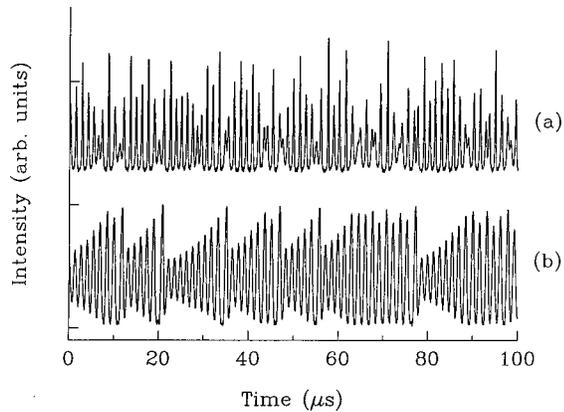
FIG. 2. Chaotic dynamics of the laser without generalized synchronization of chaos to the driving signal. The average pump-laser intensity is 0.8 W/cm^2 and the gas pressure is 19 μbar. Peak-peak pump intensity modulation strength is 28% of the mean pump intensity. (a) Chaotic laser intensity evolution; (b) evolution of the chaotic driving signal measured.

AOM, which in return transfers this modulation signal to the intensity of the pump laser beam. The undiffracted beam from the AOM is used as the pump of the $^{15}NH_3$ laser.

Effectively, this obtained pump intensity consists of a chaotically varying intensity part that acts as the chaotic driving in our experiment, and a dc intensity part that ensures that without the chaotic driving the laser is itself chaotic. The chaotic intensity of the laser is detected with a Schottky-barrier diode. To compare the chaotic laser intensity dynamics with those of the driving signal, the intensity of the diffracted beam from the AOM is monitored with a HgCdTe detector. The electronic signals from both detectors are low-noise amplified and then displayed simultaneously on a digital storage oscilloscope. A rf spectrum analyzer is also used to monitor the real-time frequency distribution of the chaotic laser output. Based on this information the frequency relationship between the chaotic driving signal and the chaotic laser dynamics is controlled through adjusting the driving signal frequency.

III. EXPERIMENTAL RESULTS

In the generalized synchronization of chaos, the dynamics of coupled chaotic systems is functionally related, and it is possible that this functional relationship is very complicated. Also in order to detect the existence of a generalized synchronization of chaos in a real physical system due to the chaotic nature of coupled dynamics, special methods are usually required. Abarbanel et al. [17] have suggested a so-called auxiliary system method to detect it in a drive-response system. The idea of this method is that instead of directly comparing the chaotic dynamics of the drive and response system, one compares the dynamics of two identical response systems subjected to the same chaotic driving. Because of the chaotic nature of the interaction and the existence of natural noise in a real physical experiment, in the absence of generalized synchronization, the chaotic dynamics of these two response systems will be unrelated. However, in the case of a generalized synchronization, their dynamics will be exactly the same. Inspired by this idea, we
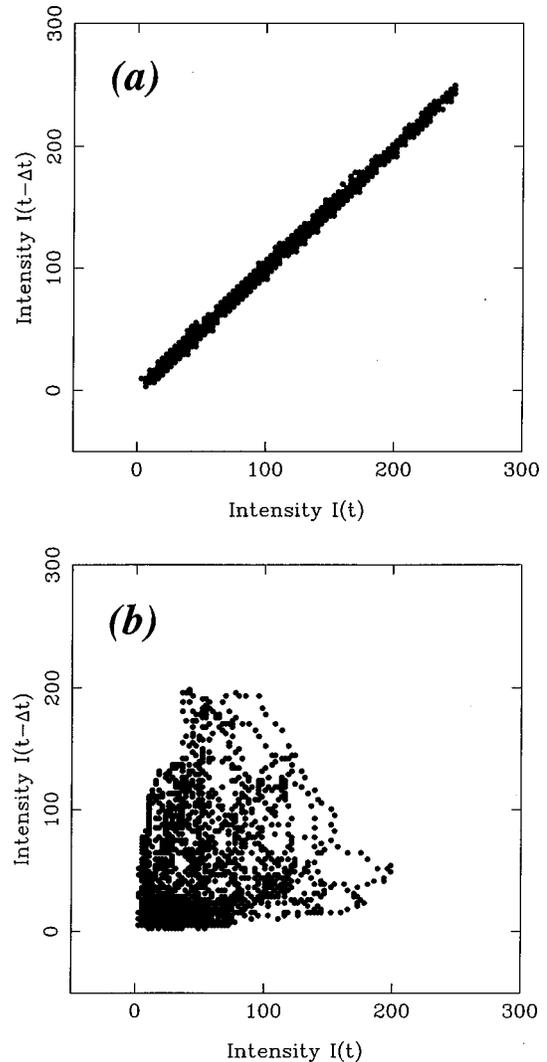

FIG. 3. Relationship plot of the laser dynamics shown in Fig. 2 under repeated drivings. Δt is the repetition period of driving signal, which is 150 μs in our experiment. (a) Relationship plot of the driving signal; (b) relationship plot of the laser dynamics.

have utilized a similar technique to detect the generalized synchronization of chaos in our system. Concretely, instead of using two response systems and comparing their dynamics, we repeatedly drive the chaotic laser under the same experimental conditions with the same chaotic wave form, and compare the chaotic dynamics of the laser under each repetition.

With this technique, the generalized synchronization of chaos in our system is easily detected. We have studied synchronization of chaos of the laser under different driving signal strengths and different frequency relationships between the driving and laser chaotic dynamics. With a fixed driving signal strength, it was observed that the average pulsation frequency separation between the laser dynamics and the driving signal plays an important role in achieving any kind of synchronization. When the average chaotic pulsation frequencies are well separated, i.e., their separation is larger than the frequency locking range described below, within our experimentally obtainable driving signal strength, no form of synchronization is observed. In this case the existence of chaotic driving makes the chaotic intensity evolution of the
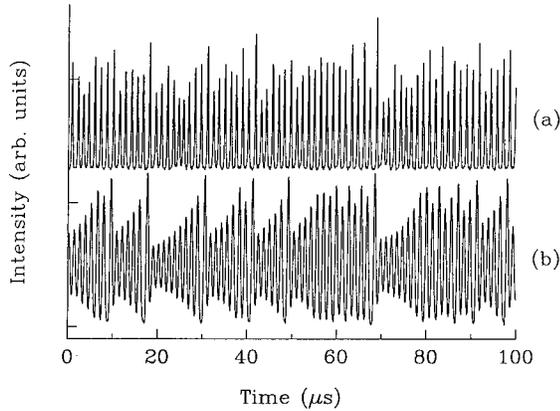
FIG. 4. Chaotic dynamics of the laser under the generalized synchronization of chaos to the driving signal. The average pump laser intensity is 1 W/cm^2 and the gas pressure is 19 μbar. Peak-peak pump intensity modulation strength is 23% of the mean pump intensity. (a) Chaotic laser intensity evolution. (b) Evolution of the chaotic driving signal measured.

laser more complicated than its original single mode chaotic dynamics. Figure 2 shows a typical experimental result observed in this situation. Figure 2(a) is the chaotic laser intensity output under the driving, and Fig. 2(b) is the chaotic driving signal, measured from the diffracted beam intensity of the AOM. We have used a chaotic wave form calculated from the Lorenz equations as our original driving signal to modulate the rf driving of the AOM. Because of the limited modulation bandwidth of the AOM this driving signal is somewhat distorted relative to true Lorenz chaos but retains the general character. Before the driving signal is switched on, the laser is in a Lorenz type of spiral chaos state [13]. Obviously, under the chaotic driving, the dynamics of the laser deviates from the Lorenz spiral chaos dynamics, and the chaotic intensity evolution of the laser is different from those of the driving signal. Figure 3(b) shows the relationship plot of the chaotic laser dynamics under repeated drivings. In our experiment the driving signal is repeated every 150 μs. The period of repetition is irrelevant to the experimental results; however, to ensure that there is no drift in experimental conditions, a relatively short period of repetition is an advantage. For comparison, we have also shown in Fig. 3(a) the relationship plot of the driving signal in the two repeated events, which shows the reliability of our detection method. Under the repeated events, the driving signal repeats itself perfectly; however, the chaotic dynamics of the laser shown in Fig. 2 does not, which indicates that the laser state is not a generalized synchronization of chaos.

When the separation between the average chaotic pulsation frequencies of the driving and the laser is small, within a certain range determined solely by the driving signal strength (with present experimental conditions, the range is about 0.15 MHz), the frequencies lock automatically together, and consequently the chaotic laser intensity pulsations become frequency entrained to the driving signal [18]. In a frequency entrained state, the chaotic dynamics of the laser is not necessarily synchronized to the driving signal. The generalized synchronization of chaos of the laser is observed in a frequency entrained state only when the driving signal strength is also beyond a certain threshold. Figure 4
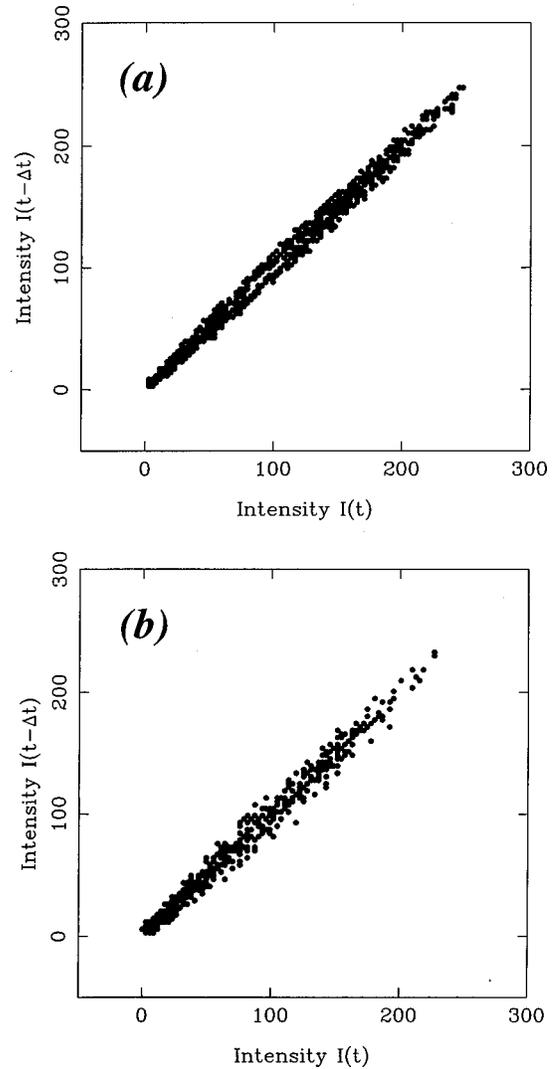

FIG. 5. Relationship plot of the laser dynamics shown in Fig. 4 under repeated drivings. Δt is the repetition period of driving signal, which is 150 μs in our experiment. (a) Relationship plot of the driving signal. (b) Relationship plot of the laser dynamics.

shows one of these experimental states of generalized synchronization of chaos. Before the chaotic driving, the laser is again in a Lorenz spiral chaos state. When the driving is turned on, after a transient, the chaotic dynamics of the laser become as shown in Fig. 4(a). In this state the average chaotic pulsation frequency of the laser is locked to that of the driving signal; however, its chaotic intensity evolutions are still different from the driving signal. Figure 5 shows the comparison of this chaotic laser dynamics under repeated drivings. Figure 5(a) is the relationship plot of the driving signal and Fig. 5(b) is that of the laser. Within the experimental error level, the chaotic dynamics of the laser repeats itself completely in repeated events, showing that its dynamics is now completely controlled by the driving signal and insensitive to the noise and initial conditions. The chaotic laser state shown in Fig. 4 is therefore an example of generalized synchronization of chaos.

The experimental result shown in Fig. 4 is recorded under a relatively weak driving strength. In this case the dynamical relation between the laser and the driving is complicated and not apparently visible. We observed experimentally that the
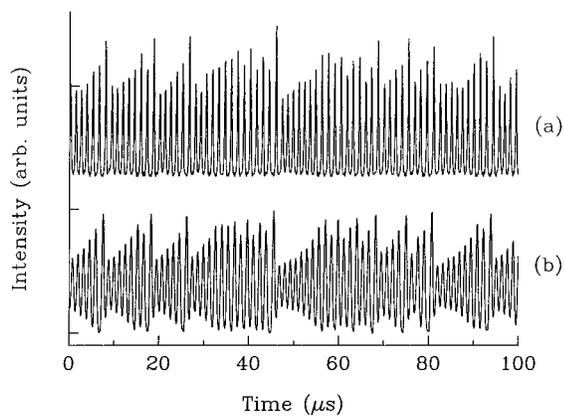
FIG. 6. Chaotic dynamics of the laser under the generalized synchronization of chaos to the driving signal. The average pump laser intensity is 1 W/cm^2 and the gas pressure is 19 μbar. Peak-peak pump intensity modulation strength is 28% of the mean pump intensity. (a) Chaotic laser intensity evolution. (b) Evolution of the chaotic driving signal measured.

functional relation in a generalized synchronization of chaos is not fixed but varies with the driving signal strength. When the driving strength is increased, the dynamical relationship between driving and response becomes simpler and simpler, and eventually they show a linear relation. Figure 6 shows another generalized synchronization of chaos observed as the driving strength is increased. In this state not only is the average chaotic pulsation frequency of the laser locked to that of the driving signal, but also the amplitudes of its pulsations become linearly related to those of the driving signal, as evidenced by the fact that they have the same spiral lengths and almost the same spiral forms. For completeness, we show in Fig. 7 the relationship plots of the laser dynamics and the driving signals in repeated events. Again they show complete repetition of the dynamics, confirming that Fig. 6 shows generalized synchronization of chaos with increased correlation to the driving signal.

IV. DISCUSSION

From our experiment we see that under a strong driving force, the chaotic dynamics of the laser can be synchronized in the generalized sense to that of the driving signal. As far as we are aware, this is the first experimental evidence of this chaotic synchronization effect in a physical system. A crucial condition of achieving this chaotic synchronization in our system is the entrainment of average chaotic pulsation frequencies between the laser and the driving. Presumably, only under this condition can the chaotic driving dynamics execute an effective influence on that of the laser. We believe that this frequency entrainment condition could be a fundamental requirement of chaotic synchronization in coupled chaotic systems.

A characteristic of the generalized synchronization of chaos is that there exists a functional relation between the dynamics of the driving and response systems. Our experimental results demonstrate that this relation is not a fixed relation but that it varies with the driving strength, and the stronger the driving strength, the simpler the relation. On the basis of our experimental results, we speculate that there
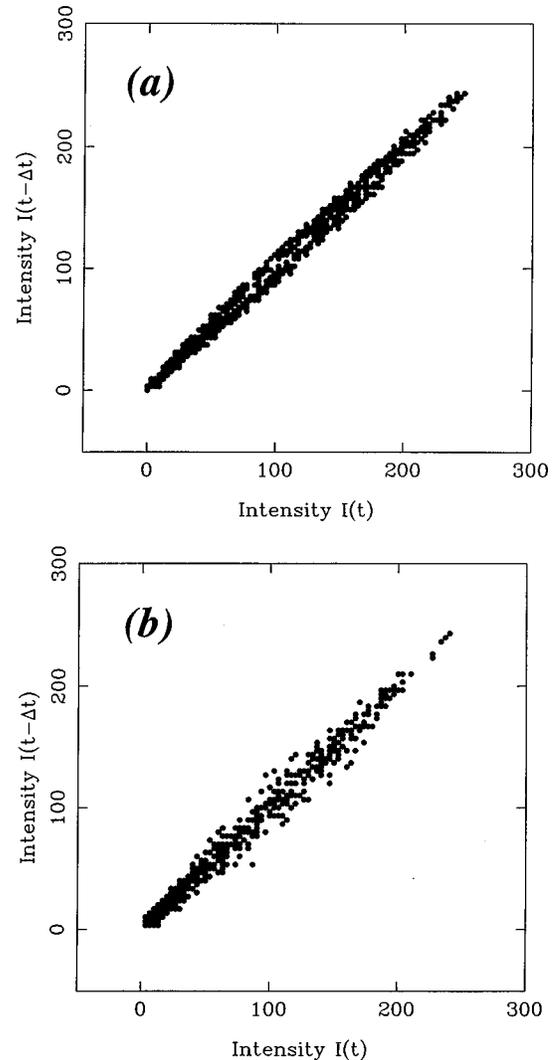

FIG. 7. Relationship plot of the laser dynamics shown in Fig. 6 under repeated drivings. Δt is the repetition period of the driving signal, which is 150 μs in our experiment. (a) Relationship plot of the driving signal. (b) Relationship plot of the laser dynamics.

exists a close relationship between the generalized synchronization and the complete synchronization of chaos. In the case of unidirectionally coupled chaotic systems, under strong driving, the chaotic nature of the response system could be suppressed, and consequently its dynamics becomes synchronized to that of the driving system. If the chaotic dynamics of the driving system is exactly the same as that of the response system (i.e., they are coupled identical systems), after suppressing the chaotic nature, the response system will immediately exhibit the driving dynamics. However, if their chaotic dynamics are not identical, after suppressing the chaotic nature, the response will not be identical but a deterministic relationship will exist between them, leading to generalized synchronization. Depending on the driving strength, this relationship changes; the stronger the driving strengths, the closer their chaotic dynamics, until eventually their dynamics become exactly the same. The complete synchronization of chaos is therefore the ultimate case of the generalized synchronization of chaos of coupled nonidentical chaotic systems. In our experiment, because of the large difference between the chaotic dynamics of the real

driving signal and the laser, and because of the limitation of the experimentally obtainable driving signal strength, we have not observed the complete synchronization of chaos of the laser. However, our experimental results certainly show an approach to this condition.

V. CONCLUSIONS

In conclusion, we have experimentally observed the generalized synchronization of chaos in a real physical system. Our experimental results show that under a nonlinear resonant interaction, the chaotic dynamics of a single mode laser can be synchronized in the sense of generalized synchronization of chaos to that of a chaotic driving signal. Under this synchronization of chaos, the chaotic dynamics of the laser is functionally related to that of the driving signal, and consequently insensitive to the noise and initial conditions. Our experimental results also show that as the coupling strength is increased, the dynamical relation between the driving signal and the laser becomes simpler and simpler, and the state of complete synchronization of chaos is approached. Our experimental results suggest that the complete synchronization of chaos is the ultimate case of the generalized synchronization of chaos in coupled nonidentical chaotic systems.

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